1-6 Reflections, Absolute Values, and Other Transformations

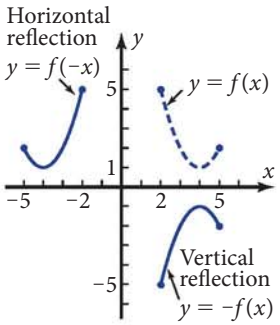

Figure 1-6a

In Section 1-3, you learned that if $y = f(x)$, then multiplying x by a nonzero constant causes a horizontal dilation. Suppose that the constant is -1. Each x-value will be $1/(-1)$ or -1 times what it was in the pre-image. Figure 1-6a shows that the resulting image is a horizontal reflection of the graph across the y-axis. The new graph is the same size and shape, simply a mirror image of the original. Similarly, a vertical dilation by a factor of -1 reflects the graph vertically across the x-axis.

In this section you will learn special transformations of functions that reflect their graphs in various ways. You will also learn what happens when you take the absolute value of a function or of the independent variable x. Finally, you will learn about odd and even functions.

Objective

Given a function, transform it by reflection and by applying *absolute value* to the function or its argument.

Reflections Across the *x*-axis and *y*-axis

Example 1 shows you how to plot the graphs in Figure 1-6a.

EXAMPLE 1 ➤ The pre-image function $y = f(x)$ in Figure 1-6a is $f(x) = x^2 - 8x + 17$, where $2 \le x \le 5$.

a. Write an equation for the reflection of this function across the y-axis.

b. Write an equation for the reflection of this function across the x-axis.

c. Plot the pre-image and the two reflections on the same screen.

SOLUTION

a. A reflection across the y-axis is a horizontal dilation by a factor of -1. So

$y = f(-x) = (-x)^2 - 8(-x) + 17$ Substitute $-x$ for x.

$y = x^2 + 8x + 17$

Domain: $2 \le -x \le 5$

$-2 \ge x \ge -5$ or $-5 \le x \le -2$ Multiply all three sides of the inequality by -1. The inequalities reverse.

b. $y = -f(x)$ For a reflection across the x-axis, find the opposite of $f(x)$.

$y = -x^2 + 8x - 17$ The domain remains $2 \le x \le 5$.